

REVISÃO PARA PROVA 2

1. Classifique e resolva, se possível, o sistema linear a seguir pelo método de Gauss-Jordan.

(a) $\begin{cases} x + y - 2z = 7 \\ 2x - y - z = 1 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 2 & -1 & -1 & 1 \end{array} \right] \xrightarrow{L_2 = L_2 + L_1(-2)} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & -3 & 3 & -13 \end{array} \right] \xrightarrow{L_3/(-3)} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & 1 & -1 & 13/3 \end{array} \right] \xrightarrow{L_1 = L_1 - L_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 8/3 \\ 0 & 1 & -1 & 13/3 \end{array} \right] \quad \left\{ \begin{array}{l} C_C = 2 = C_I \Rightarrow \text{compatível} \\ n = 3 \Rightarrow \text{indeterminado} \end{array} \right.$$

$$\left\{ \begin{array}{l} x - z = 8/3 \Rightarrow x = 8/3 + z \\ y - z = 13/3 \Rightarrow y = 13/3 + z \end{array} \right. \quad \left\{ \begin{array}{l} z = 0 \\ x = 8/3 \\ y = 13/3 \end{array} \right. \quad \begin{array}{l} \text{é uma solução} \\ z = 0 \end{array}$$

(b) $\begin{cases} x + y - 2z = 0 \\ 2x + 2y - 4z = 1 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 2 & 2 & -4 & 1 \end{array} \right] \xrightarrow{L_2 = L_2 + L_1(-2)} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \left\{ \begin{array}{l} C_C = 2 \\ C_I = 1 \end{array} \right. \Rightarrow \text{incompatível}$$

2. Calcule os valores de x e y para que os vetores \vec{u} e \vec{v} sejam iguais.

(a) $\vec{u} = (2x - 1, 4y + 5)$ e $\vec{v} = (5, 1)$

$$\left\{ \begin{array}{l} 2x - 1 = 5 \Rightarrow 2x = 5 + 1 \Rightarrow x = \frac{6}{2} \Rightarrow x = 3 \\ 4y + 5 = 1 \Rightarrow 4y = 1 - 5 \Rightarrow y = \frac{-4}{4} \Rightarrow y = -1 \end{array} \right.$$

(b) $\vec{u} = (1, 17)$ e $\vec{v} = (-x + 4, 5y - 3)$

$$\left\{ \begin{array}{l} -x + 4 = 1 \Rightarrow x = 4 - 1 \Rightarrow x = 3 \\ 5y - 3 = 17 \Rightarrow 5y = 17 + 3 \Rightarrow y = \frac{20}{5} \Rightarrow y = 4 \end{array} \right.$$

(c) $\vec{u} = (5x, 4)$ e $\vec{v} = (15, y^2)$

$$\left\{ \begin{array}{l} 5x = 15 \Rightarrow x = \frac{15}{3} \Rightarrow x = 3 \\ y^2 = 4 \Rightarrow y = \pm\sqrt{4} \Rightarrow y = \pm 2 \end{array} \right.$$

3. Determinar:

(a) $2\vec{u} - \frac{1}{5}\vec{v}$, sendo $\vec{u} = (4, 1)$ e $\vec{v} = (5, -15)$

$$2(4, 1) - \frac{1}{5}(5, -15) = (8, 2) - (1, -3) = (7, 5)$$

(b) $\frac{1}{2}\vec{u} - 2\vec{v} - \vec{w}$, sendo $\vec{u} = (2, -4, 1)$, $\vec{v} = (1, -1, 0)$ e $\vec{w} = (-1, 1, -1)$

$$\frac{1}{2}(2, -4, 1) - 2(1, -1, 0) - (-1, 1, -1) = \left(1, -2, \frac{1}{2}\right) - (2, -2, 0) - (-1, 1, -1) = \left(0, -1, \frac{3}{2}\right)$$

(c) $\vec{u} - 2\vec{v} + \vec{w}$, sendo $\vec{u} = 2\vec{i} - \vec{j} + 3\vec{k}$, $\vec{v} = \vec{i} - 2\vec{j}$ e $\vec{w} = 5\vec{j} - \vec{k}$

$$\vec{u} - 2\vec{v} + \vec{w} = (2, -1, 3) - 2(1, -2, 0) + (0, 5, -1) = (2, -1, 3) - (2, -4, 0) + (0, 5, -1) = (0, 8, 2) = 8\vec{j} + 2\vec{k}$$

(d) $\vec{u} - \vec{v} + 3\vec{w}$, sendo $\vec{u} = \vec{i} - \frac{1}{2}\vec{j} + \vec{k}$, $\vec{v} = \vec{i} - \frac{2}{3}\vec{j}$ e $\vec{w} = \vec{i} - \vec{k}$

$$\left(1, -\frac{1}{2}, 1\right) - \left(1, -\frac{2}{3}, 0\right) + 3(1, 0, -1) = \left(0, \frac{1}{6}, 1\right) + (3, 0, -3) = \left(3, \frac{1}{6}, -2\right) = 3\vec{i} + \frac{1}{6}\vec{j} - 2\vec{k}$$

4. Sejam $A(-1, -1)$, $B(1, 1)$ e $C(-1, 3)$ vértices consecutivos do paralelogramo ABCD.

(a) Encontre as coordenadas do vértice D.

$$\begin{aligned} \overrightarrow{AD} = \overrightarrow{BC} &\Rightarrow D - A = C - B \Rightarrow (x, y) - (-1, -1) = (-1, 3) - (1, 1) \Rightarrow (x + 1, y + 1) = (-2, 2) \\ &\Rightarrow \begin{cases} x + 1 = -2 \\ y + 1 = 2 \end{cases} \Rightarrow x = -3, y = 1 \Rightarrow D(-3, 1) \end{aligned}$$

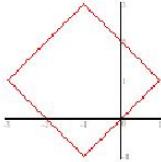
(b) Calcule a área do paralelogramo.

$$\begin{aligned} |\overrightarrow{AB}| &= |B - A| = \|(1, 1) - (-1, -1)\|_2 = \|(2, 2)\|_2 = \sqrt{8} \\ |\overrightarrow{BC}| &= |C - B| = \|(-1, 3) - (1, 1)\| = \|(-2, 2)\|_2 = \sqrt{8} \Rightarrow \text{área} = (\sqrt{8})^2 = 8 \end{aligned}$$

(c) Utilize o produto escalar para verificar que o paralelogramo é um quadrado.

É quadrado pois $\overrightarrow{AB} \cdot \overrightarrow{BC} = (2, 2) \cdot (-2, 2) = 0$, isto é, $\overrightarrow{AB} \perp \overrightarrow{BC}$ e os lados são iguais.

(d) Desenhe o paralelogramo.



5. Escrever \vec{w} como combinação linear dos demais vetores.

(a) $\vec{u} = (3, -1)$, $\vec{v} = (4, 5)$ e $\vec{w} = (2, -7)$

$$(2, -7) = a(3, -1) + b(4, 5) = (3a + 4b, -a + 5b) \Rightarrow \begin{cases} 3a + 4b = 2 \\ -a + 5b = -7 \end{cases} \Rightarrow a = 2, b = -1 \Rightarrow \boxed{\vec{w} = 2\vec{u} - 2\vec{v}}$$

(b) $\vec{u} = -\vec{i} + 2\vec{j}$, $\vec{v} = 3\vec{i} - 4\vec{j}$ e $\vec{w} = 8\vec{i} - 12\vec{j}$

$$(8, -12) = a(-1, 2) + b(3, -4) = (-a + 3b, 2a - 4b) \Rightarrow \begin{cases} -a + 3b = 8 \\ 2a - 4b = -12 \end{cases} \Rightarrow a = -2, b = 2 \Rightarrow \boxed{\vec{w} = -2\vec{u} + 2\vec{v}}$$

(c) $\vec{u} = \vec{i} + 2\vec{j} - \vec{k}$, $\vec{v} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{t} = -\vec{i} + \vec{j} - 2\vec{k}$ e $\vec{w} = -5\vec{i} + 3\vec{j} - 4\vec{k}$

$$(-5, 3, -4) = a(1, 2, -1) + b(2, -1, 1) + c(-1, 1, -2) = (a + 2b - c, 2a - b + c, -a + b - 2c)$$

$$\left\{ \begin{array}{l} a + 2b - c = -5 \\ 2a - b + c = 3 \\ -a + b - 2c = -4 \end{array} \right. \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & -5 \\ 2 & -1 & 1 & 3 \\ -1 & 1 & -2 & -4 \end{array} \right] \xrightarrow{L_2 = L_2 + L_1(-2)} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -5 \\ 0 & -5 & 3 & 13 \\ 0 & 3 & -3 & -9 \end{array} \right] \xrightarrow{L_3 = L_3 + L_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -5 \\ 0 & -5 & 3 & 13 \\ 0 & 0 & -6 & 4 \end{array} \right] \xrightarrow{L_{23}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & -2 & -2 \end{array} \right] \xrightarrow{L_3 = L_3 / (-2)} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{L_1 = L_1 + L_3(-1)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right] \Rightarrow \left\{ \begin{array}{l} a = 0 \\ b = -2 \\ c = 1 \end{array} \right. \Rightarrow \boxed{\vec{w} = -2\vec{v} + \vec{t}}$$

(d) $\vec{u} = (2, 5, 1)$, $\vec{v} = (-1, 2, 1)$, $\vec{t} = (3, -4, 0)$ e $\vec{w} = (5, 15, 1)$

$$(5, 15, 1) = a(2, 5, 1) + b(-1, 2, 1) + c(3, -4, 0) = (2a - b + 3c, 5a + 2b - 4c, a + b)$$

$$\left\{ \begin{array}{l} 2a - b + 3c = 5 \\ 5a + 2b - 4c = 15 \\ a + b = 1 \end{array} \right. \quad \left[\begin{array}{ccc|c} 2 & -1 & 3 & 5 \\ 5 & 2 & -4 & 15 \\ 1 & 1 & 0 & 1 \end{array} \right] \xrightarrow{L_{13}} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 5 & 2 & -4 & 15 \\ 2 & -1 & 3 & 5 \end{array} \right] \xrightarrow{L_2 = L_2 + L_1(-5)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 7 & -9 & 10 \\ 2 & -1 & 3 & 5 \end{array} \right] \xrightarrow{L_3 = L_3 + L_1(-2)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 7 & -9 & 10 \\ 0 & -3 & 5 & 5 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 7 & -9 & 10 \\ 0 & -3 & 5 & 5 \end{array} \right] \xrightarrow{L_2 / (-3)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & -9/3 & -10/3 \\ 0 & -3 & 5 & 5 \end{array} \right] \xrightarrow{L_1 = L_1 - L_2} \left[\begin{array}{ccc|c} 1 & 0 & 9/3 & 13/3 \\ 0 & 1 & -9/3 & -10/3 \\ 0 & -3 & 5 & 5 \end{array} \right] \xrightarrow{L_3 = L_3 + L_2(3)} \left[\begin{array}{ccc|c} 1 & 0 & 9/3 & 13/3 \\ 0 & 1 & -9/3 & -10/3 \\ 0 & 0 & 1 & -7 \end{array} \right] \xrightarrow{L_3 / 7} \left[\begin{array}{ccc|c} 1 & 0 & -4/3 & 13/3 \\ 0 & 1 & 4/3 & -10/3 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4/3 & 13/3 \\ 0 & 1 & 4/3 & -10/3 \\ 0 & 0 & 1 & -1 \end{array} \right] \xrightarrow{L_1 = L_1 + L_3(4/3)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right] \xrightarrow{L_2 = L_2 + L_3(-4/3)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$$\Rightarrow a = 3, b = -2, c = -1 \Rightarrow \boxed{\vec{w} = 3\vec{u} - 2\vec{v} - \vec{t}}$$