

CÁLCULO II - AULA 02: REGRAS BÁSICAS DE DERIVAÇÃO

Derivada de Constante: $(k)' = 0 \quad k \text{ constante}$

Derivada de Função Potência: $(x^n)' = n \cdot x^{n-1} \quad (\text{se } n \neq -1)$

Derivada de Constante vezes Função: $[kf(x)]' = kf'(x)$

Derivada da Soma e Diferença: $[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$$(\sqrt{2})' = 0$$

$$(x^5)' = 5x^{(5-1)} = 5x^4$$

$$\left(\frac{1}{x^4}\right)' = (x^{-4})' = -4x^{-5} = -\frac{4}{x^5}$$

$$\text{Exemplos: } (x)' = (x^1)' = 1x^0 = 1$$

$$(\sqrt{x})' = (x^{1/2})' = \frac{1}{2\sqrt{x}}$$

$$(2x^4)' = 2 \cdot (x^4)' = 2 \cdot (4 \cdot x^3) = 8x^3$$

$$\left(\sqrt[3]{x^2}\right)' = (x^{2/3})' = 2/3x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

$$(6 + 3x^5 - x^7)' = (6)' + (3x^5)' - (x^7)' = 15x^4 - 7x^6$$

Exercício 1 Calcule a derivada das funções dadas:

$$1. \ f(x) = 3x^4 - 5x^2 + 1$$

$$2. \ f(s) = \sqrt{3}(s^3 - s^2)$$

$$3. \ f(x) = 5x^\pi$$

$$4. \ f(x) = 5x^4 - 2tx^2 + t^2$$

$$5. \ f(t) = 5x^4 - 2tx^2 + t^2$$

$$6. \ v(r) = \frac{4}{3}\pi r^3$$

$$7. \ f(x) = \frac{9x^4}{4}$$

$$8. \ f(x) = \left(\frac{3}{5}\right)x^{15}$$

$$9. \ f(t) = \frac{1}{4}t^4 - \frac{1}{2}t^2$$

$$10. \ f(x) = x^{3/2}$$

$$11. \ f(x) = \frac{3}{5}x^{5/3}$$

$$12. \ f(x) = 2x^{1/2}$$

$$13. \ f(x) = x^4 - 5 + x^{-2} + 4x^{-4}$$

$$14. \ f(x) = x^{-2/3}$$

$$15. \ g(x) = \frac{3}{x^2} + \frac{5}{x^4}$$

$$16. \ f(x) = 2x^2\sqrt{x}$$

$$17. \ f(x) = \frac{1}{\sqrt{x}}$$

$$18. f(x) = \frac{x^2}{\sqrt[3]{x^2}}$$

$$19. f(x) = \frac{x^{-7/8}}{\sqrt{x}}$$

$$20. f(y) = \sqrt[5]{y^4} + x^3$$

$$21. f(x) = (2x - 1)(3x^2 + 2)$$

DERIVADA DO PRODUTO E DO QUOCIENTE DE FUNÇÕES

Se f e g são deriváveis em x , tem-se

$$\begin{cases} (f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x) \text{ ou } (uv)' = u'v + uv' \\ \left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \text{ ou } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \end{cases}$$

Exemplo 1 $[(x^2 - 1)(x + 2)]' = 2x(x + 2) + (x^2 - 1) = 2x^2 + 4x + x^2 - 1 = 3x^2 + 4x - 1$

De fato, $\frac{d}{dx}(x^2 - 1)(x + 2) = \frac{d}{dx}(x^3 + 2x^2 - x - 2) = 3x^2 + 4x - 1$

Exemplo 2 $\left(\frac{x^2 + x - 2}{x - 1}\right)' = \frac{(2x + 1)(x - 1) - (x^2 + x - 2)}{(x - 1)^2} = \frac{(2x^2 - x - 1) - (x^2 + x - 2)}{(x - 1)^2} = \frac{x^2 - 2x + 1}{(x - 1)^2}$
 $= \frac{(x - 1)^2}{(x - 1)^2} = 1.$ De fato, $\frac{x^2 + x - 2}{x - 1} = \frac{(x + 2)(x - 1)}{x - 1} = x + 2 \Rightarrow \left(\frac{x^2 + x - 2}{x - 1}\right)' = (x + 2)' = 1$

Exercício 2 Calcule a derivada, utilizando as regras do produto e/ou quociente:

$$1. f(x) = (2x - 1)(3x^2 + 2)$$

$$2. g(x) = (2x^2 + 5)(4x - 1)$$

$$3. f(x) = (2x^4 - 1)(5x^3 + 6x)$$

$$4. g(x) = (4x^2 + 3)^2$$

$$5. f(x) = (x^2 - 3x + 2)(2x^3 + 1)$$

$$6. y = \left(\frac{1}{x^2} + 5x^4\right)\left(x - \frac{2}{x^3}\right)$$

$$7. f(x) = \frac{x}{x - 1}$$

$$8. h(x) = \frac{5x}{1 + 2x^2}$$

$$9. F(y) = \frac{2y + 1}{3y + 4}$$

$$10. H(x) = \frac{x^2 + 2x + 1}{x^2 - 2x + 1}$$

$$11. f(x) = \frac{x^3 - 8}{x^3 + 8}$$

$$12. g(x) = \frac{x^4 - 2x^2 + 5x + 1}{x^4}$$

$$13. f(x) = \frac{2x + 1}{x + 5} (3x - 1)$$

RESPOSTAS

Exercício 1

1 $f'(x) = 12x^3 - 10x$

2 $f'(s) = \sqrt{3} (3s^2 - 2s)$

3 $f'(x) = 5x^{\pi-1}\pi$

4 $f'(x) = 20x^3 - 4tx$

5 $f'(t) = -2x^2 + 2t$

6 $v'(r) = 4\pi r^2$

7 $f'(x) = 9x^3$

8 $f'(x) = 9x^{14}$

9 $f'(t) = t^3 - t$

10 $f'(x) = \frac{3}{2}\sqrt{x}$

11 $f'(x) = \sqrt[3]{x^2}$

12 $f'(x) = \frac{1}{\sqrt{x}}$

13 $f'(x) = 4x^3 - 2x^{-3} - 16x^{-5}$

14 $f'(x) = -\frac{2}{3\sqrt[3]{x^5}}$

15 $g'(x) = -\frac{6}{x^3} - \frac{20}{x^5}$

16 $f'(x) = 5(\sqrt{x})^3$

17 $f'(x) = -\frac{1}{2(\sqrt{x})^3}$

18 $f'(x) = \frac{4}{3}\sqrt[3]{x}$

19 $f'(x) = -\frac{11}{8\sqrt[8]{x^{19}}}$

20 $f'(y) = \frac{4}{5\sqrt[5]{y}}$

21 $f'(x) = 18x^2 - 6x + 4$

Exercício 2

1 $f'(x) = 18x^2 - 6x + 4$

2 $g'(x) = 24x^2 - 4x + 20$

3 $f'(x) = 70x^6 + 60x^4 - 15x^2 - 6$

4 $g'(x) = 64x^3 + 48x$

5 $f'(x) = 10x^4 - 24x^3 + 12x^2 + 2x - 3$

6 $y' = -\frac{1}{x^2} + 25x^4 + \frac{10}{x^6} - 10$

7 $f'(x) = -\frac{1}{(x-1)^2}$

8 $h'(x) = \frac{5-10x^2}{(1+2x^2)^2}$

9 $F'(y) = \frac{5}{(3y+4)^2}$

10 $H'(x) = -\frac{4(x+1)}{(x-1)^3}$

11 $f'(x) = \frac{48x^2}{(x^3+8)^2}$

12 $g'(x) = \frac{4x^2 - 15x - 4}{x^5}$

13 $f'(x) = \frac{6(x^2 + 10x + 1)}{(x+5)^2}$